El Monte Union High School District

Course Outline

High School Rosemead High School
Rev. 1/18/2017

Title: Integrated Math 1 Lab (IM1 Lab)

Transitional*_______(Eng. Dept. Only)

Sheltered (SDAIE)*___Bilingual*___

AP**________Honors**__________

Department: Mathematics

Grade Level (s): 9/10

Semester___________Year X

Year of State Framework Adoption _______

This course meets graduation requirements:

( ) English
( ) Fine Arts
( ) Foreign Language
( ) Health & Safety
( ) Math
( ) Physical Education
( ) Science
( ) Social Science
(X) Elective

Department/Cluster	Approval Date

*Instructional materials appropriate for English language learners are required.

**For AP/Honors course attach a page describing how this course is above and beyond a regular course. Also, explain why this course is the equivalent of a college level class.

1. **Prerequisite(s):**
   a. 9th or 10th grade student
   b. C or Below grade in 8th grade Math class
   c. Math SBAC Proficiency level: Standards Nearly Met or Standards Not Met
   d. 200-500 Quantile* score – Tier 1 (3rd – 5th Grade level skills)
   e. 501-800 Quantile* score – Tier 2 (6th – 8th Grade level skills)
   ***Quantile score will be obtained through Scholastic Math Inventory (SMI)**

   Corequisite(s):
   Integrated Math 1 or equivalent OR SUP Integrated Math 1A/1B or equivalent

2. **Short description of course which may also be used in the registration manual:**

   The Integrated Math 1 Lab is designed to narrow student learning gap and support the development of student skills in (a) *concepts and procedures* – students apply mathematical concepts and procedures, (b) *problem solving and modeling/data analysis* – students use appropriate tools and strategies to solve real world and mathematical problems and (c) *communicate reasoning* – students demonstrate ability to support mathematical conclusions. This course also serves as a support class for Integrated Math 1 (IM1). The primary functions of the course are to develop deeper understanding, strengthen reasoning and analytical skills and support conceptual and mathematical foundations to accelerate and develop academic language and content in mathematics. *Students in Integrated Math 1 Lab must also be enrolled in IM 1 with the same instructor if* *possible*. This course follows a specific curriculum for Intervention that supports the Integrated Math 1 core curriculum, and will focus on various targeted mathematical concepts and skills.

Teachers must implement tiered instruction/intervention during Integrated Math 1 Lab instructional time. The instructional focus for students with Quantile score of 200-500 will be 3rd to 5th grade level focus standards, while students with Quantile score of 501-800 will be 6th to 8th grade level focus standards. (See Curricular Map and Instructional Focus for Intervention Matrix).

3. **Describe how this course integrates the schools SLOs (School-wide Learning Outcomes):**

   a. Academic Achievers: Students will further develop reading and writing skills.
   b. Critical Thinkers: Students will use critical thinking skills in their reading analysis and their various writing assignments.
   c. Technology Competent Users: Students will use technology to research topics and create essays.
   d. Ethical, Respectful Individuals: Students will be respectful while working in diverse collaborative groups.
   e. Active Community Participants: Students will develop skills that will increase their ability to participate in the community.

4. **Describe the additional efforts/teaching techniques/methodology to be used to meet the needs of English language learners:**

   a. Oral and academic language development will be utilized.
   b. Study skills and Cornell notes will be emphasized.
   c. RESEARCH BASED strategies and activities such as SIOP, AVID, metacognitive strategies, Marzano strategies, and Kinsella strategies will assist student learning.
   d. Prior knowledge will be used to build connections and support new learning.
   e. Vocabulary and content development will be highlighted.
   f. Graphic organizers, visuals, realia, audio, and technology software will be utilized during instruction in order to support multiple learning modalities and Universal Design for learning.
   g. Multiple teaching models will be utilized: Concept Attainment Model (CAM), Concrete-Pictorial-Abstract Model (CPAM), Explicit Direct Instruction
   h. Engagement routines such as think-write-pair-share, text mark-up, and group and paired work.
   i. Writing support scaffolds such as sentence-framing and paragraph-framing will be utilized.
   j. Reasoning and justifying answers will be highly encouraged.
   k. Flexible instructional organization for whole-class, group, paired and individualized learning will be implemented.

5. **Describe the interdepartmental articulation process for this course:**

   Interdisciplinary articulation is ongoing and driven by a common need to improve mathematical competency skills school-wide. Continuous collaboration with the Science department will be implemented to reinforce application and utilization of mathematical skills across content areas.

6. **Describe how this course will integrate academic and vocational concepts, possibly through connecting activities. Describe how this course will address work-based learning/school to career concepts:**

   Connections will be drawn between skills taught and practiced in this course to applications in various careers and to college readiness. Problem solving application and performance tasks will be emphasized. Students will be taught fundamental career skills such as reasoning, communicating, analyzing data, modeling, and interpretation and solving mathematical problem.

7. Materials of Instruction (Note that materials of instruction for English language learners are required and should be listed below.)

a. Textbook(s) and Core Reading(s): *Math 180 Course 1 and Course 2: Revolutionary Math Intervention for Algebra and Beyond*

b. Supplemental Materials and Resources:

- *Kanold, T.D., Burger, E.B., Dixon, J.K., Larson, M.R., Leinwand, S.J. (2015). Integrated Mathematics, Volume 1 and 2. Houghton Mifflin Harcourt Publishing Company.*
- *HMH Revolutionary Math Intervention for Algebra and Beyond*
- *On Core Mathematics Deluxe Examview CD-ROM Grade 6-12*
- *Houghton Mifflin Harcourt on Core Mathematics Activity Generator CD-ROM*
- *HMH Integrated Math 1 California Teacher Edition with Solutions*
- *HMH Integrated Math 1 California Student Workbooks (Volume 1 and 2)*
- *HMH Integrated Math 1 California response to Intervention Teacher Resource Blankline Master*
- *HMH Integrated Math 1 California Online Teacher Resource Management Center (myhmh.com)*

8. Objectives of Course (Including Projects, Activities, & Duration of Units); Reference to State Frameworks; Student Performance Standards; Evaluation/Assessment/Rubrics: Minimal Attainment for Student to Pass.

a. **Objective of the course:** Develop and rebuild conceptual and mathematical foundations, deeper understanding and proficiency of computational skills, and strengthen reasoning and analytical skills.

- Students will receive support to master the California Common Core Standards for Integrated Mathematics 1 with emphasis on conceptual understanding, procedural/computational skills and problem solving skills.
- Students will apply the Mathematical Practice Standards through out the course with specific content standards for Integrated Mathematics 1.
- Students will experience mathematics as a coherent, useful, and logical subject that makes use of the ability to make sense of problem situation.
- Students will review and revisit previous mathematics content that will help them formalize and extend the mathematics learned in middle school.
- Students will be supported in identifying their critical areas of need in Mathematics.

b. **Unit details including projects, activities and duration of units (pacing) plan:**

Regular daily lessons will align with the pacing of the Integrated Math 1 Lab that supports the Integrated Math 1 curriculum. Curriculum pacing and individual unit plans indicate a tiered alignment of content clusters that will support students and decrease the achievement gap. Performance tasks and problem solving applications will be provided to ensure that students have opportunities to synthesize and apply the learned mathematical concepts and computational skills (see attached Integrated Math 1 Lab Curriculum Map and Unit Plans).

c. **Indicate references to stat framework(s)/standards (If state standard is not applicable then national standard should be used.)**

All standards can be found in the Houghton, Mufflin, and Harcourt Integrated Mathematics Volume 1 and Volume 2 and are outlined in the EMUHSD Curricular Map for Integrated Math 1. (See Houghton, Mufflin, and Harcourt (HMH) Standard Lesson File).

d. Student Performance standards:

Students will participate in class discussions, complete classroom and home assignments and assessments. The following is the grading scale used for overall performance: 90%-100% - A; 80%-89% - B; 70%-79% - C; 60%-69% - D; and Below 60% - F.

Because the purpose of the Integrated Math 1 Lab course is to support IM1 core course. A student's Integrated Math 1 Lab and Integrated Math 1 grades should not vary by more than one LETTER grade.

e. Evaluation/Assessment/Rubrics

Participation, assignments, classwork, unit test, performance task, and benchmarks scores from the core class will be used for assessment purposes in Integrated Math 1 Lab.

f. Include minimal attainment for student to pass course

Students must attain an average of 60% in all coursework, and must pass the Integrated Math 1 course in order to receive a passing grade in the Integrated Math 1 Lab. The course grade and the Integrated Math 1 course grade should not vary by more than one LETTER grade. Each of the following account for 25% of the grade respectively: Homework, classwork/participation, Assessment (including Quantile Score), Projects/Performance Task

Math Bridge Lab 1: Curricular Map and Instructional Focus for Intervention Matrix

3rd	4th	5th	6th	7th	8th	Integrated Math 1 Content Focus
Represent and solve problems involving multiplication and division	Use four operations to solve problems	Understand the place value system	Apply and extend previous understandings of multiplication and division to divide fractions by fractions	Apply and extend previous understanding of operations with fractions to add, subtract, and divided rational numbers	Work with radical and integer exponents	Unit 1 (Relationship Between Quantities) • Reason quantitatively and use units to solve problems • Interpret the structure of expressions • Create equations that describe numbers or relationships.
Understand properties of multiplication and the relationship to division	Generalize place value understanding for multi-digit numbers	Perform operations with multi-digit whole numbers and decimals to the hundredths	Apply and extend previous understandings of numbers to the system of rational numbers	Analyze proportional relationships and use them to solve real-world and mathematical problems	Understand the connections between proportional relationships, lines, and linear equations	Unit 2 (Linear and Exponential Relationships) • Represent and solve equations and inequalities graphically • Understand the concept of a function and use function notation • Interpret functions that arises in applications in terms of a context • Analyze functions using different representation • Build new functions from existing functions • Construct and compare linear, quadratic, and exponential models and solve problems. • Interpret expressions for functions in terms of the situation they model.
Multiply & divide within 100	Use place value understanding and properties to perform multi-digit arithmetic	Use equivalent fractions as a strategy to add and subtract fractions	Understand ratio concepts and use ratio reasoning to solve problems	Use properties of operations to generate equivalent expressions	Analyze and solve linear equations and pairs of simultaneous linear equations	Units 3 (Reasoning with Equations0 • Understand solving equations as a process of reasoning and explain the reasoning • Solve equations and inequalities in one variable • Solve systems of equations
Solve problems involving the four operations, & identify & explain patterns in arithmetic	Extend understanding of fraction equivalence and ordering	Apply and extend previous understandings of multiplication and division to multiply and divide fractions	Apply and extend previous understanding of arithmetic to algebraic expression	Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Define, evaluate, and compare functions	Unit 4 (Descriptive Statistics) • Summarize, represent, and interpret data on a single count or measurement variable • Summarize, represent, and interpret data on two categorical and quantitative variables • Interpret linear models
Develop understanding of fractions as numbers	Build fractions from unit fractions	Understand concept of volume and relate volume to multiplication and addition	Reason about and solve one-variable equations and inequalities		Use functions to model relationship between quantities	Unit 5 (Congruence, Proof, and Construction) • Experiment with transformations in the plane • Understand congruence in terms of rigid motions • Make geometric constructions
Solve problems	Understand decimal	Graph points in the	Represent and			Unit 6 (Connecting Algebra and Geometry

involving measurement and estimation of time, volume, & mass	notation fro fractions, and compare decimal fractions	coordinate plane to solve real-world and mathematical problems	analyze quantitative relationships between dependent and independent variables			through coordinates) • Use coordinates to prove simple geometric theorems algebraically.
Understand concepts of area and relate area to multiplication and to addition						

Tier 1 Topic clusters of Intervention: 200-500 Quantile Score (Grade 3-5 Focus Topics/Clusters)

Tier 2 Topic clusters of Intervention: 501-800 Quantile Score (Grade 6-8 Focus Topics/Clusters)

The 8 mathematical Practices	HMH Integrated Math 1
#1 Make sense of problems and persevere in solving them. #2 Reason abstractly and quantitatively. #3 Construct viable arguments and critique the reasoning of other. #4 Model with mathematics. #5 Use appropriate tools strategically. #6 Attend to precision. #7 Look for and make use of structure. #8 Look for and express regularity in repeated reasoning.	Unit 1: Quantities and Modelling Unit 2: Understanding of Functions Unit 3: Linear Functions, Equations, and Inequalities Unit 4: Statistical Models Unit 5: Linear Systems and Piecewise-Define Functions Unit 6: Exponential Relationships Unit 7: Transformations and Congruence Unit 8; Lines, Angles, and Triangles Unit 9: Quadrilaterals and Coordinate Proofs

Sources:

Kanold, T.D., Burger, E.B., Dixon, J.K., Larson, M.R., Leinwand, S.J. (2015). Intgerated Mathematics Volume 1. Houghton Mifflin Harcourt Publishing Company.

Common Core State Standard Appendix A0

Houghton Mifflin Harcourt
Integrated Math I
Scope and Sequence & CCSS Standards Alignment

First Semester – 77 *Instructional* days

Unit 1: Understanding Quantities and Expressions (10 Instructional days)

N-Q Quantities

Reason quantitatively and use units to solve problems. (*Supporting Cluster*)

N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*

N-Q.2 Define appropriate quantities for the purpose of descriptive modeling.*

N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*

A-SSE Seeing Structure in Expressions

Interpret the structure of expressions (*Major Cluster)*

A-SSE.1 Interpret expressions that represent a quantity in terms of its context.*

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*

N-RN The Real Number System

Use properties of rational and irrational numbers. (*Additional Cluster)*

N-RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Note: N-RN 1 and 2 are in the **Algebra 2** curriculum.

Unit 1 Notes:
This initial unit starts with a treatment of quantities (numbers with units) as preparation for work with computation, modeling and functions. Throughout the course emphasis should be placed on reasoning and sense making rather than calculation and symbolic manipulation. Linear and non-linear examples should be explored in this unit as well as Mathematical Modeling (as a conceptual Category theme) is introduced to create context to the problems of the unit.

A short treatment of the CCSS general notion of a "quantity" thought of as *a number with a specific unit of measure,* should unfold.

Include unit analysis – (such as Factor-Label or Unit-Factor Methods) with connections to simple science examples too. Examples should include simple quantities with standard units of measure; and the fundamental dimensions of quantities such as length, time, weight, and temperature.

Division of quantities – Examples of quantities with quotient units such as speed, flow rate, frequency, price, density, pressure; and Flow rates such as a river moving at 3000ft/sec. Thus quotient units as "rates"; quotient units and unit conversion; and unit analysis is included here.

Multiplication of quantities and products of units such as area and volume as examples of quantities with product units; person-days and kilowatt hours as other examples of product units.

Note, in this initial unit there is *very limited* equation solving: Mental Math only. The primary focus is on further development and understanding of the grades 6-8 structure of expressions as well as an understanding of the properties of rational numbers.

This Unit builds on 8.NS 1 *Know that there are numbers that are not rational, and approximate them by rational numbers* and 7.EE1-2 *Use properties of operations to generate equivalent expressions.*

Unit 2: Understanding Functions (15 Instructional days)

F-IF Interpreting Functions

Understand the concept of function and use function notation (*Major Cluster*)

F-IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

F-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$

Interpret functions that arise in applications in terms of the context (*Major Cluster*)

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

Analyze functions using different representations (*Supporting Cluster*)

F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph one [quadratic] function and an algebraic expression for another, say which has the larger maximum. Note: comparison of simple functions only — In this unit you would compare the graph of one function with the table of another, and compare values of the function for given points

Unit 2 Notes:
In this unit there should be a general treatment of the function concept with minimal use of symbolic expressions at this stage. Instead you should place emphasis on the idea of a function as a mapping represented in tables or graphs and possibly simple algebraic expressions.

The functions used in this unit, should be mostly simple linear and simple exponential. In Unit 3 and Unit 7 students will more extensively study linear and exponential functions; respectively. They are introduced here in order to compare two different types of functions. Simple quadratic functions could also be used to illustrate the properties of functions as well, if desired.

Properties should include: Domain and range; functions defined by graphs and their interpretation; functions defined by tables and their interpretation; properties of particular functions (rate of change, zeros) and their meaning in an application; sums and differences of two functions; product of a function and a constant; simple equations defined in terms of functions and their solution – such as finding the domain value or values for a given range value; sequences and functions defined recursively.

This unit builds on the Grade 8 CCSS Domain for Functions, 8.F 1, 8.F 2, 8.F 3 *Functions: Define, evaluate, and compare functions.*
and 8.F 4 and 8.F 5 *Functions: Use functions to model relationships between quantities.*

This unit should also provide a continued student experience with the Mathematical Modeling aspects of the course – using real life data to describe or generate the functions.

Unit 3: Linear Functions (17 Instructional days)

F-IF Interpreting

Interpret functions that arise in applications in terms of the context (*Major Cluster*)

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. (Linear only)

Analyze functions using different representations *(Supporting Cluster)*

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated (Linear and absolute Value only) functions

a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one [quadratic] function and an algebraic expression for another, say which has the larger maximum.

Note: Linear only — (In this unit you would compare the graph of one linear function with the table of another, and compare values of the function for given points)

F-BF Building Functions

Build a function that models a relationship between two quantities (*Supporting Cluster*)

F-BF.1 Write a function that describes a linear relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

*F-LE Linear, Quadratic, and Exponential Models**

Construct and compare linear, [quadratic], and exponential models and solve problems (*Supporting Cluster)* (Note: Linear aspect only)

F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

Unit 3 Notes:
A thorough treatment of linear functions of one variable in the general form $f(x) = mx + b$ *with* representation of linear functions using expressions, graphs, and tables.

Students identify and interpret the three parameters of x-intercept, y-intercept and rate of change or slope m from any of the three representations.

Students Build and create expressions for linear functions using the expressions or parameters of $f(x) = mx+b$ *and* $f(x) = m(x-x0)$

Students demonstrate understanding and can explain geometrically why the graph is a line;

Understanding ***constant*** rate of change is the unique feature of linear functions.

Thus students should model a variety of situations using linear functions; look at the special properties of linear functions such as $y = mx$ and their role in representing proportional relationships; and connect to the idea of linear sequences ("arithmetic" sequences) as a linear function model. Students should conclude this Chapter working with the absolute value function $f(x) = |mx + b|$.

Note: This unit builds on 8.EE 5 and 8.EE 6 *Expressions and Equations: Understand the connections between proportional relationships, lines, and linear equations* and 8.F 2 and 8.F 3 *Functions: Define, evaluate, and compare functions. Use functions to model relationships between quantities.*

Unit 4: Modeling Linear Data (18 Instructional days)

*S-ID Interpreting Categorical and Quantitative Data**

Interpret linear models (*Major Cluster*)

S-ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.*

S-ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.*

S-ID.9 Distinguish between correlation and causation.*

Summarize represent and interpret data on two categorical and quantitative variables (*Supporting Cluster)*

S-ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.*

S-ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.*

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, models*.

b. Informally assess the fit of a function by plotting and analyzing residuals.

c. Fit a linear function for a scatter plot that suggests a linear association.

Note: SD 1, 2, and 3 below are considered additional clusters and not tested and could go into this unit as a combined standard or as supporting material…. SD.4 is now an Algebra II only Standard.

Summarize, represent, and interpret data on a single count or measurement variable (*Additional Cluster)*

S-ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).*

S-ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.*

S-ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).*

S-ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.*

Unit 4 Notes:
This unit is the statistical connection within Algebra 1. It can be placed here as Unit 4 or it could be exchanged with Unit 5 if desired. However, applications n Unit 5 and 6 can be more rich if this unit is done here.

The unit builds upon statistics from grades 6 and 8 and includes understanding the single variable categorical and quantitative data sets using plots, graphs, measures of center and spread if you choose to address S-ID. 1-4 which although considered part of an additional cluster may good as part of a review form grades 6-8.

The unit is really about the connections to Linear Functions from Unit 3 using two variable data sets. The unit introduces statistical tools to investigate bi-variate data that include frequency tables, scatterplots, and lines that fit the data.

Students interpret, describe and summarize the data to make determinations regarding linear and non-linear associations, clustering, outliers, positive, negative and no correlations. The treatment continues by using linear functions and correlation coefficients (using technology such as a TI-84+, etc.) to make inferences and solve various modeling problems in the context of the data sets.

The students learn to distinguish between correlation and causation, and will do similar statistical work for exponential functions (Unit 7) and Quadratic function (Unit 8) later.

Unit 5: Linear Equations and Inequalities in ONE Variable

(17 instructional days – End of first Semester)

*A-CED Creating Equations**

Create equations that describe numbers or relationships (*Major Cluster*)

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from* ***linear*** *[and quadratic] functions, and [simple rational and exponential functions.]**

A-CED.3 Represent constraints by equations or inequalities, [and by systems of equations and/or inequalities,] and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.**

A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.**

A-REI Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning (*Major Cluster*)

A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable *(Major Cluster)*

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Represent and solve equations and inequalities graphically

A-REI.11 Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are **linear,** [polynomial, rational,] **absolute value,** [exponential, and logarithmic functions.]*

A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), [and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.]

Unit 5 Notes:
Based on the established understanding of linear functions in a plane from Unit 2 and3, this Unit provides various methods for finding the solution of linear equations in **one unknown** such as $ax + b = cx + d$, with an extension to solution of linear inequalities such as $ax + b < c$

Connecting linear equations to linear functions is a shift in teaching algebra 1 and *signals a priority* that provides and allows for visual and data representations to an understanding of equations.

This unit also develops student understanding that the solution of a linear equation $ax + b = cx + d$ involves the intersection of the graphs of two linear functions – as solving linear equations both through manipulation of expressions and graphically is of critical importance. Thus, students can now consider equations as an ***equivalence of two functions*.**

Understanding the conditions under which a linear equation has no solution, one solution, or an infinite number of solutions can now be understood graphically as well.

In this unit students understand the solution x to a linear equation as the number where two linear functions have the same value and learn how to solve a linear equation step by step analytically.

This unit also expects students to solve linear inequalities such as $ax + b < c$ and $|ax + b| < c$, and represent the solution on a number line. One solution method (perhaps using technology) could include the comparison of the graphs of the functions represented by each side of the inequality.

This unit includes writing (or creating) Linear Equations to solve a wide variety of problems in context **(See Cluster with A-CED 1 and 3).**

This unit builds on 8.EE 7
Expressions and Equations: Analyze and solve linear equations [and pairs of simultaneous linear equations.]

This Unit signifies the end of First Semester.

Second Semester – 78 Instructional days

Unit 6: Linear Equations and Inequalities in Two Variables (15 Instructional days)

*A-CED Creating Equations**

Create equations that describe numbers or relationships (*Major Cluster*)

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

A-CED.3 Represent constraints [by equations or inequalities, and] by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*.*

A-REI Reasoning with Equations and Inequalities

Solve systems of equations (*Additional Cluster*)

A-REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically (*Major Cluster*)

A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A-REI.12 [Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality),] and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Unit 6 notes:
This unit presents a thorough treatment of linear equations in two unknowns such as $ax + by + c = 0$. This includes simultaneous solution of two such equations.

Students demonstrate an understanding of the differences among the solutions to a single equation in one unknown, solutions to a single equation in two unknowns, and "simultaneous" solutions to two equations in two unknowns.

Students demonstrate understanding of the solution to an equation $ax + by + c = 0$ in two unknowns as the intersection of the graph of this function with the x-y plane and make the analogy with functions and equations of one variable/unknown. Consider exploration of the symmetric form $(x/x0) + (y/y0) = 1$ of a linear equation in two unknowns.

Students also solve problems (and do additional mathematical modeling) set in an applied context using systems of equations and their solutions. This unit "feels" like a current unit for system of equations, but as an emphasis on modeling and building the systems as well.

Unit 7: Understanding Exponential functions, Models and Equations (20 Instructional days)

F-LE Linear, Quadratic, and Exponential Models[*]

Construct and compare linear, [quadratic,] and exponential models and solve problems (*Supporting Cluster)*

F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).[*]

F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.[*]

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unity interval relative to another.

F-LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, [quadratically,] or (more generally) as polynomial function.

Interpret expressions for functions in terms of the situation they model (*Supporting Cluster*)

F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.[*]

F-BF Building Functions

Build a function that models a relationship between two quantities (*Supporting Cluster*)

F-BF.1 Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential. (Note: This standard is not assessed in PARCC grade 9).

Build new functions from existing functions (*Additional Cluster*)

F-BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

F-IF Interpreting Functions

Analyze functions using different representations (*Supporting Cluster*)

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated (exponential only).

F-IF.8a Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of a function.

F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one [quadratic] function and an algebraic expression for another, say which has the larger maximum.

Interpret functions that arise in applications in terms of the context (*Major Cluster*)

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; [relative maximums and minimums; symmetries; end behavior; and periodicity.]**

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. (*Percent rate of change)*

A-SSE Seeing Structure in Expressions

Write expressions in equivalent forms to solve problems (*Supporting Cluster*)

A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*

c. Use the properties of exponents to transform expression for exponential functions. For example, the expression 1.15t can be rewritten as (1.151/12)12t=1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

S-ID Interpreting Categorical and Quantitative Data*

Summarize represent and interpret data on two categorical and quantitative variables (*Supporting Cluster)*

S-ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.*

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize* ***linear and exponential*** *models.*

b. Informally assess the fit of a function by plotting and analyzing residuals

Unit 7 Notes:
This Unit connects to Expressions and Equations 8.EE *Work with radicals and integer exponents.*1. *Know and apply the properties of integer exponents to generate equivalent numerical expressions.* This unit is PACKED with content, and thus needs the 20 days to unfold the clusters of comparing, building and interpreting exponential functions.

The unit provides a deep examination of exponential functions, the Laws of exponents: Including definition of exponent notation; sum law for exponents; product law for exponents; definition of negative exponent notation and basic characteristics of exponential functions.

The unit also extends ideas and understandings from Unit 3 on linear functions to the parallel ideas for exponential functions with special emphasis on the notion of ***constant percent*** rate of change (as compared to constant rate of change in Chapter 3) – using repeated multiplication as the big idea, geometric sequences and recursive definitions; the meaning of the dependent variable; the meaning of the independent variable; parameters and their meanings; ways of measuring amount of growth via the constant difference (linear) and constant ratio (exponential).

The unit also features the comparison of linear and exponential functions in a context and from various representations of those types of functions. Statistics is brought back into this unit via fitting an exponential function to data.

Unit 8: Geometric Foundations and Tools (10 Instructional days)

G-CO Congruence

Experiment with transformations in the plane (*Supporting Cluster*)

G-CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc

Make geometric constructions (*Supporting Cluster*)

G-CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

G-CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Unit 9: Congruence and Rigid Transformations (15 Instructional days)

G-CO Congruence

Experiment with transformations in the plane (*Supporting Cluster*)

G-CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G-CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G-CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G-CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions (*Major Cluster*)

G-CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

Unit 9 Notes:

Building upon the informal experiences with basic geometric objects and relationships from 8th grade, the goal in this Unit is to increase the precision and the use of the definitions established in Unit 8.

The emphasis in this unit should be on the role of definitions and communicating mathematical explanations and arguments (with precision) that cause and preserve congruence, rather than on developing a deductive axiomatic system. In Grade 8 rigid motions were explored, but in this course they must be more precisely defined, and their properties explored.

This unit should provide the detail necessary to use transformations as a "Proving" tool throughout future Units - especially in course II. G-CO.6 takes the time needed to develop understanding and fluency about congruence of different shapes based on the rigid motions of rotation, reflection, and translation.

G-CO.7 and 8 (Triangle congruence and similarity) could be added to this unit. However, they are intentionally left out of the unit and placed in Course II as part of the triangle work there.

Unit 10: Connecting algebra and geometry (18 Instructional days)

G-CO Congruence

Prove geometric theorems (*Major Cluster*)

G-CO.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints*.

G-GPE Expressing Geometric Properties with Equations

Use coordinates to prove simple geometric theorems algebraically (*Major Cluster*)

G-GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

G-GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

G-GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, [prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle;] prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

Unit 10 Notes:

This unit establishes Coordinate Geometry (and some proof) using both slope and distance formula. Much of the *Geometric System* at this stage will verified using three methods: Constructions (or some such tool – including *dynamic* software), Rigid Motions from Unit 9, or Coordinate Geometry (A major cluster area). This allows for all three methods then, to be used throughout the rest of the Integrated mathematics courses – especially with Triangles in course II and Circles in course III.

This final course I unit brings together many of the concepts taught in course I. The emphasis should be a focus on the mathematical practice of making viable arguments and critiquing the reasoning of others (#3). Multiple representations (coordinates, synthetic, and algebraic proofs of properties can be analyzed and compared) and the lines and angles theorems explored… thus leading to the opportunity for multiple solution pathways.

Note too, the special properties to be developed for understanding as revealed in the Standards for this unit—including work with parallel lines and transversals, essential to the Triangle Sum Theorem in the Course II.

At this stage in the course students are armed with three different methods for verifying the properties learned in Course I as true and applying various properties that will surface again as part of course II.

Course I Notes:

In an integrated pathway for the high school standards, there is a desire to ensure an introduction into all of the Conceptual Categories of the high school Standards. In a way that ties them together s part of a connected experience for the student.

To do so, requires addressing key content standard clusters from number and quantity, algebra, geometry, functions, statistics and expressions/equations. In an Integrated Course one, I believe it makes most sense sequentially, to unfold these connections across the Conceptual Categories and across the Domains (especially the clusters) as follows:

1) First establish issues arising from Number and Quantity, connecting to the 8th grade outcomes, and provide a base for all student work in the rest of the course.

2) Next, establish deep student understanding of function - including treatment of linear and exponential functions, the two primary "types" of change for study in this course.

3) Third proceed to connecting these first two conceptual categories to work with expressions and equations (think of equations as an equivalence of two functions), by using all three modes

for creating and solving linear equations in one variable and then two variables: Numerical (tables, etc.), Visual (Graphical) and analytical (algebraic).

4) Fourth, this sets up a discussion of quantitative data that is linear and the potential for an integrated statistics unit. This discussion is best embedded earlier before the deeper discussion regarding exponential functions and equations.

5) Fifth, and finally, the opportunity for rich geometric connections ends this course. Not only can this work integrate and extend coordinate geometry and functions connections from earlier Course I units, these final three units also establish a preview of deeper work in Course II.

Finally, *Modeling* is viewed best in the entire Integrated Pathway literally. Meaning an integrated student learning experience should be built into every unit. Thus, modeling is not viewed as a product resulting from a single chapter or unit, best viewed as a process – a way of life for students in every part and aspect of the course.